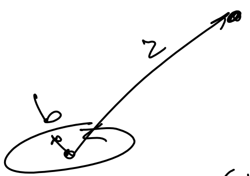
PHYS 100C, Lecture 11

Tuesday, May 05, 2009
8:41 PM

* Oscillating Magnetic Dipole:



Loop radius b, oscillating current, producin magnetic dipole:

$I = I_0 \cos(\omega t)$ $\vec{m} = \pi b^2 I \cdot \hat{z} = m_0 \cdot \cos\omega t \, \hat{z}$

Where $m_0 \equiv \pi b^2 I_0$

Write $A(r,t)$, expand $\frac{1}{r}$ term and time retardation term $t - r/c$ with respect to loop assuming:

$b \ll r$ (far away from dipole)

$b \ll \frac{c}{\omega} = \frac{1}{k} = \frac{\lambda}{2\pi}$ (loop smaller than λ)

$\frac{c}{\omega} = \frac{\lambda}{2\pi} \ll r$ (wavelength smaller than r)

Very, very similar approach to el. dipole from last lecture.

Bottom-line result:

$$\langle S \rangle_{MAG} = \frac{\mu_0 m_0^2 \omega^4}{32\pi^2 c^3} \cdot \frac{\sin^2\theta}{r^2} \cdot \hat{r}$$

Again, very similar to el. dipole case,

$$\langle S \rangle_{el} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \cdot \frac{\sin^2\theta}{r^2} \cdot \hat{r}$$

Ratio of the two: $\frac{\langle S \rangle_{MAG}}{\langle S \rangle_{el}} = \left(\frac{m_0}{p_0 c}\right)^2$

If dimensions & parameters are similar:

$p_0 \equiv qd$ $m_0 \equiv \pi b^2 I_0$

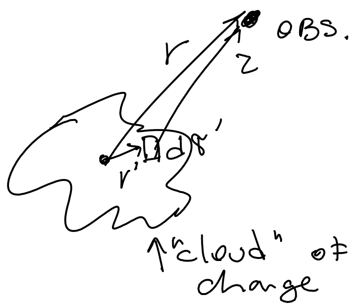
$d = \pi b$, $I = q \cdot \omega$,

$$\frac{\langle S \rangle_{mag}}{\langle S \rangle_{el}} = \left(\frac{\cancel{\pi} b^2 \cancel{q} \omega}{\cancel{\pi} \cancel{q} \cdot b \cdot c} \right)^2 = \left(\frac{\omega b}{c} \right)^2 = \left(\frac{b \cdot 2\pi}{\lambda} \right)^2$$

But we said $b << \lambda$, so

$$\frac{\langle S \rangle_{mag}}{\langle S \rangle_{el}} << 1$$

* General derivation of radiation



$$V(r,t) = \frac{1}{4\pi\varepsilon_0} \cdot \int \frac{\rho(r', t - \mathscr{r}/c)}{\mathscr{r}} d\tau'$$

$$\mathscr{r} = \sqrt{r^2 + r'^2 - 2\bar{r} \cdot \bar{r}'}$$

Assume $r' << r$
(far away from "cloud")

expand w.r. to 1^{st} order correction:

$$\mathscr{r} = r \sqrt{1 + \underbrace{\left(\frac{r'}{r}\right)^2}_{2^{nd} \text{ order}} - \underbrace{2 \frac{\bar{r} \cdot \bar{r}'}{r^2}}_{1^{st} \text{ order}}} \approx r \left(1 - 2 \frac{\bar{r} \cdot \bar{r}'}{r^2} \right)^{1/2} \approx r \left(1 - \frac{\bar{r} \cdot \bar{r}'}{r^2} \right)$$

$$1/\mathscr{r} = \frac{1}{r} \left(1 + \frac{\bar{r} \cdot \bar{r}'}{r^2} \right)$$

Time-retarded component can be expanded around
$t_0 \equiv t - \frac{r}{c}$ (retarded time at $r = 0$)

$$t = t - \mathscr{r}/c = \underbrace{t - \frac{r}{c}}_{t_0} + \underbrace{\left(\frac{\bar{r} \cdot \bar{r}'}{r^2} \right) \cdot \frac{\mathring{r}}{c}}_{\Delta t}$$

$$\rho(r', t) = \rho(r', t_0) + \dot{\rho}(r', t_0) \cdot \Delta t + \ldots$$

Where we "forget" higher order
Taylor expansion terms $\frac{1}{2} \ddot{\rho} (\Delta t)^2$, etc.

Taylor expansion terms $\frac{1}{2}\ddot{\rho}(\delta t)^2$, etc.

Note $\delta t = \left(\frac{\vec{r}\cdot\vec{r}'}{r^2}\right)\cdot\frac{r}{c} = \frac{\vec{r}\cdot\vec{r}'}{|r|\cdot c} = \frac{\hat{r}\cdot\vec{r}'}{c}$

$$V(r,t) = \frac{1}{4\pi\varepsilon_0}\int\left[\frac{\rho+\dot{\rho}\,\delta t}{r}\left(1+\frac{\vec{r}\cdot\vec{r}'}{r^2}\right)\right]\cdot d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0 r}\left[\int\rho\cdot d\tau' + \frac{\hat{r}}{r}\cdot\int r'\rho\cdot d\tau' + \right.$$

$$\left. + \frac{\partial}{\partial t}\int r'\rho\cdot d\tau'\right]$$

where ρ is $\rho(r', t_0)$

But $\int\rho\cdot d\tau' = Q$ (total charge, not time-dependent!)

$\int\vec{r}'\rho\cdot d\tau = \vec{p}$ (dipole)

$$V(r,t) = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{Q}{r} + \frac{\hat{r}\cdot\vec{p}}{r^2} + \frac{\hat{r}\cdot\dot{\vec{p}}}{rc}\right]$$

$$A(r,t) = \frac{\mu_0}{4\pi}\int\frac{J(r', t-\frac{\mathscr{r}}{c})}{\mathscr{r}}\,d\tau \approx \frac{\mu_0}{4\pi}\int\frac{J\cdot d\tau}{\mathscr{r}}$$

$$\int J\cdot d\tau' = \frac{\partial p}{\partial t} \Rightarrow A(r,t) = \frac{\mu_0}{4\pi}\frac{\dot{p}}{r}$$

When calulating ∇V for E, need to keep only terms that scale $\sim 1/\mathscr{r}$, and time-dependent, which is $\frac{\hat{r}\cdot\dot{p}}{rc}$

Note ∇f has terms like $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial t_0}\cdot\frac{\partial t_0}{\partial x}$

So that: $\nabla f = \dot{f}\,\nabla t_0$

But $\nabla t_0 = \nabla(t - r/c) = -\frac{\nabla r}{c} = -\frac{\hat{r}}{c}$

So $\nabla V = \frac{1}{4\pi\epsilon_0}\cdot\left[\frac{\hat{r}\cdot\ddot{p}}{rc}\right]\nabla t_0 = -\frac{1}{4\pi\epsilon_0 c^2}\cdot\frac{\hat{r}\cdot\ddot{p}}{r}\cdot\hat{r}$

$\nabla\times A$ has terms $(\nabla\times\dot{p})$: $\frac{\partial \dot{p}_x}{\partial y}\cdot\hat{z} = \underbrace{\frac{\partial \dot{p}_x}{\partial t_0}}_{\ddot{p}}\cdot\frac{\partial t_0}{\partial y}\cdot\hat{z}$

$\nabla\times A = \frac{\mu_0}{4\pi r}\cdot[\overbrace{\nabla t_0}^{\hat{r}}\times\ddot{p}] = -\frac{\mu_0}{4\pi rc}[\hat{r}\times\ddot{p}]$

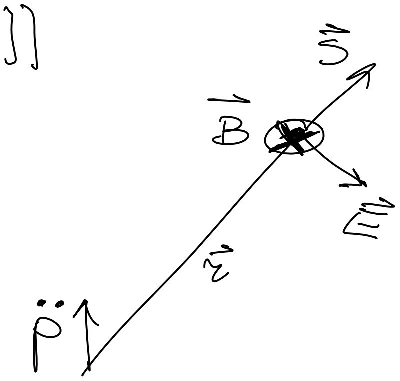
$\frac{\partial A}{\partial t} = \frac{\mu_0}{4\pi}\cdot\frac{\ddot{p}}{r}$ OR $\hat{r}\cdot\hat{r} = 1 \Rightarrow \frac{\partial A}{\partial t} = \frac{\mu_0}{4\pi}\cdot\frac{\ddot{p}\,\hat{r}\cdot\hat{r}}{r}$

Use BAC-CAB rule backwards for E:

$\vec{E} = -\nabla V - \frac{\partial A}{\partial t} = \frac{\mu_0}{4\pi r}[\hat{r}\times[\hat{r}\times\ddot{p}]]$

$\vec{B} = -\frac{\mu_0}{4\pi rc}\cdot[\hat{r}\times\ddot{p}]$

$\vec{S} = \frac{1}{\mu_0}(E\times B) = \frac{\mu_0(\ddot{p})^2}{16\pi^2 c^2}\left(\frac{\sin^2\theta}{r^2}\right)\hat{r}$



Note: we did something very similar last lecture, if

$p = p_0\cdot\cos\omega t \Rightarrow \ddot{p} = -p_0\omega^2\cdot\cos\omega t$

and you can get $\langle S\rangle = \frac{\mu_0 p_0^2\cdot\omega^4}{32\pi^2 c^2}\cdot\frac{\sin^2\theta}{r^2}\cdot\hat{r}$

Total power over $4\pi r^2$

$P = \int \vec{S}\cdot d\vec{a} = \frac{\mu_0(\ddot{p})^2}{6\pi c}$

Note $\vec{E}\perp\vec{B}$, $\vec{E},\vec{B}\perp\hat{r}$,

$E/B = c$ (EM wave)

Next on PHYS-100C:

GARBAGE TRUCK & FLIES!